ALLOWABLE STRESS DESIGN OF CONCRETE MASONRY BASED ON THE 2012 IBC & 2011 MSJC

INTRODUCTION

Concrete masonry elements can be designed by using one of several methods in accordance with the *International Building Code* (IBC, ref. 2) and, by reference, *Building Code Requirements for Masonry Structures* (MSJC Code, ref. 1): allowable stress design, strength design, direct design, empirical design, or prestressed masonry. This TEK provides a basic overview of design criteria and requirements for concrete masonry assemblies designed using allowable stress design provisions. For masonry design in accordance with the strength design, prestressed or empirical provisions, the reader is referred to TEK 14-04B, *Strength Design Provisions for Concrete Masonry* (ref. 5), TEK 14-20A, *Post-Tensioned Concrete Masonry Wall Design* (ref. 8), and TEK 14-08B, *Empirical Design of Concrete Masonry Walls* (ref. 4), respectively.

The content presented in this edition of TEK 14-07C is based on the requirements of the 2012 *International Building Code* (ref. 2a), which in turn references the 2011 edition of the MSJC Code (ref. 1a).

Significant changes were made to the allowable stress design (ASD) method between the 2009 and 2012 editions of the IBC. In previous codes, the IBC included alternative load combinations for ASD, and the MSJC ASD criteria allowed a one-third increase in allowable stresses for load combinations that include wind or seismic. The one-third stress increase is not included in the 2011 MSJC. In addition, previous code versions allowed the use of strength-level load combinations in ASD to compensate for the lack of service-level load combinations in previously referenced versions of ASCE 7, *Minimum Design Loads for Buildings and Other Structures* (ref. 3). Currently, however, ASCE 7-10 includes both service level and strength level load combinations, so this "pseudo-strength" procedure is no longer included in the current ASD method.

This TEK provides a general review of the pertinent allowable stress design criteria contained within the 2011 MSJC. Allowable stress design is based on the following design principles and assumptions:

- Within the range of allowable stresses, masonry elements satisfy applicable conditions of equilibrium and compatibility of strains.

- Stresses remain in the elastic range.

- Plane sections before bending remain plane after bending. Therefore, strains in masonry and reinforcement are directly proportional to the distances from the neutral axis.

- Stress is linearly proportional to strain within the allowable stress range.

- For unreinforced masonry, the resistance of the reinforcement, if present, is neglected.

- For reinforced masonry design, all tensile stresses are resisted by the steel reinforcement. Masonry in tension does not contribute to axial or flexural strength.

- The units, mortar, grout, and reinforcement, if present, act compositely to resist applied loads.

Based on these assumptions, the internal distribution of stresses and resulting equilibrium is illustrated in Figure 1 for unreinforced masonry and Figure 2 for reinforced masonry.

Using allowable stress design, the calculated design stresses on a masonry member (indicated by lowercase f) are compared to code-prescribed maximum allowable stresses (indicated by a capital F). The design is acceptable when the calculated applied stresses are less than or equal to the allowable stresses ($f \leq F$).

DESIGN LOADS

Utilizing ASD, masonry elements are sized and proportioned such that the anticipated service level loads can be safely and economically resisted using the specified material strengths. The specified strength of masonry and reinforcement are in turn reduced by appropriate safety factors. Minimum design

loads for allowable stress design are included in ASCE 7-10, *Minimum Design Loads for Buildings and Other Structures*, or obtained from the IBC.

UNREINFORCED MASONRY

For unreinforced masonry, the masonry assembly (units, mortar, and grout if used) is designed to carry all applied stresses (see Figure 1). The additional capacity from the inclusion of reinforcing steel, such as reinforcement added for the control of shrinkage cracking or prescriptively required by the code, is neglected. Because the masonry is intended to resist both tension and compression stresses resulting from applied loads, the masonry must be designed to remain uncracked.

Unreinforced Out-of-Plane Flexure

Allowable flexural tension values as prescribed in the 2011 MSJC Code vary with the direction of span, mortar type, bond pattern, and percentage of grouting as shown in Table 1. For assemblies spanning horizontally between supports, the code conservatively assumes that masonry constructed in a bond pattern other than running bond cannot reliably transfer flexural tension stresses across the head joints. As such, the allowable flexural tension values parallel to the bed joints (perpendicular to the head joints) in these cases are assumed to be zero. In cases where a continuous section of grout crosses the head joint, such as would occur with the use of open-ended units or bond beam units with recessed webs, tension resisted only by the minimum cross-sectional area of the grout may be considered.

Because the compressive strength of masonry is much larger than its corresponding tensile strength, the capacity of unreinforced masonry subjected to net flexural stresses is almost always controlled by the flexural tension values of Table 1. For masonry elements subjected to a bending moment, M, and a compressive axial force, P, the resulting flexural bending stress is determined using Equation 1.

$$f_b = \frac{Mt}{2I_n} - \frac{P}{A_n}$$

Eqn. 1

CMU-TEC-002-23, *Weights and Section Properties of Concrete Masonry Assemblies,* (ref. 6) provides typical values for the net moment of inertia, I_n, and cross-sectional area, A_n, for various wall sections. If the value of the bending stress, f_b, given by Equation 1 is positive, the masonry section is controlled by tension and the limiting values of Table 1 must be satisfied. Conversely, if f_b as given by Equation 1 is negative, the masonry section is in compression and the compressive stress limitation of Equation 2 must be met.

$$f_b \leq F_b = {}^1\!/_3\, f'_m$$

Eqn. 2

Unreinforced Axial Compression and Flexure

While unreinforced masonry can resist flexural tension stresses due to applied loads, unreinforced masonry is not permitted to be subjected to net axial tension, such as that due to wind uplift on a roof connected to a masonry wall or the overturning effects of lateral loads. While compressive stresses from dead loads can be used to offset tensile stresses, reinforcement must be
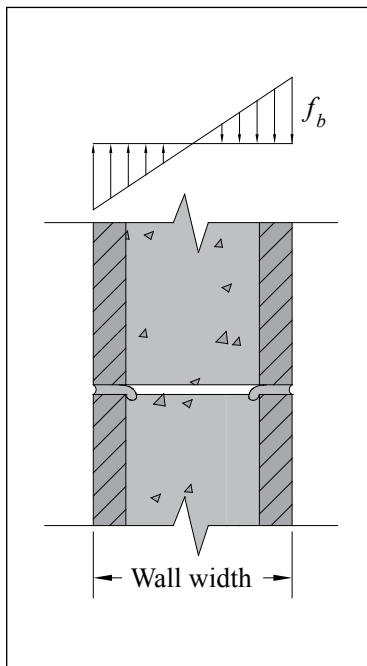

Figure 1—Unreinforced Masonry Stress Distribution

2A: Neutral axis within the compression face shell
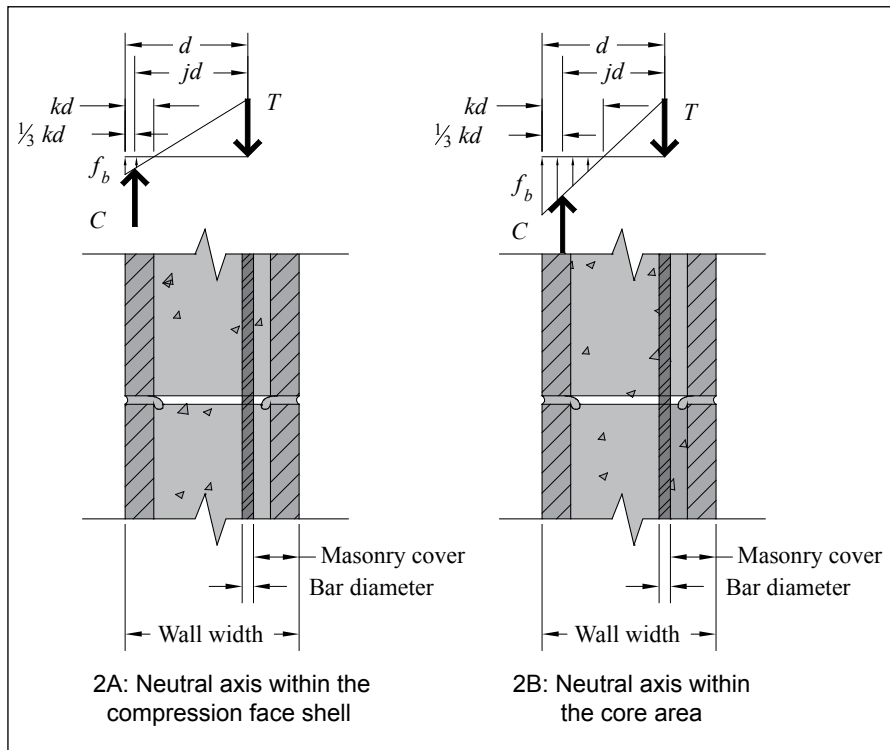
2B: Neutral axis within the core area

Figure 2—Reinforced Masonry Stress Distribution

incorporated to resist the resulting tensile forces when the element is subject to a net axial tension.

When masonry elements are subjected to compressive axial loads only, the calculated compressive stress due to applied load, f_a, must not exceed the allowable compressive stress, F_a, as given by Equations 3 or 4, as appropriate.

For elements having $h/r \le 99$:

$$f_a \le F_a = \frac{1}{4} f'_m \left[1 - \left(\frac{h}{140r} \right)^2 \right] \quad \text{Eqn. 3}$$

For elements having $h/r > 99$:

$$f_a \le F_a = \frac{1}{4} f'_m \left(\frac{70r}{h} \right)^2 \quad \text{Eqn. 4}$$

A further check for stability against an eccentrically applied axial load is included with Equation 5, whereby the axial compressive load, P, is limited to one-fourth the buckling load, P_e. With Equation 5, the actual eccentricity of the applied load, e, is used to determine P_e. Moments on the assembly due to loads other than the eccentric load are not considered in Equation 5.

$$P \le \frac{1}{4} P_e = \frac{1}{4} \left(\frac{\pi^2 E_m I_n}{h^2} \right) \left(1 - 0.577 \frac{e}{r} \right)^3 \quad \text{Eqn. 5}$$

When unreinforced masonry elements are subjected to a combination of axial load and flexural bending, a unity equation is used to proportion the available allowable stresses to the applied loads per Equation 6. This check ensures that the critical sections remain uncracked under design loads.

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \le 1 \quad \text{Eqn. 6}$$

Unreinforced Shear

Shear stresses on unreinforced masonry elements are calculated using the net cross-sectional properties of the masonry in the direction of the applied shear force using the following relation:

$$f_v = \frac{VQ}{I_n b} \quad \text{Eqn. 7}$$

Equation 7 is applicable to determining both in-plane and out-of-plane shear stresses. Because unreinforced masonry is designed to remain uncracked, it is not necessary to perform a cracked section analysis to determine the net cross-sectional area of the masonry.

Table 1—Allowable Flexural Tensile Stresses, psi (kPa) (ref. 1a)

Direction of flexural tensile stress and masonry type	Mortar types			
	Portland cement/ lime or mortar cement		Masonry cement or air-entrained portland cement/lime	
	M or S	N	M or S	N
Normal to bed joints: Solid units Hollow units[A] Ungrouted	53 (366)	40 (276)	32 (221)	20 (138)
Fully grouted	33 (228) 86 (593)	25 (172) 84 (579)	20 (138) 81 (559)	12 (83) 77 (531)
Parallel to bed joints in running bond: Solid units Hollow units	106 (731)	80 (552)	64 (441)	40 (276)
Ungrouted & partially grouted	66 (455)	50 (345)	40 (276)	25 (172)
Fully grouted	106 (731)	80 (552)	64 (441)	40 (276)
Parallel to bed joints in masonry not laid in running bond: Continuous grout section parallel to bed joints	133 (917)	133 (917)	133 (917)	133 (917)
Other	0 (0)	0 (0)	0 (0)	0 (0)

[A] For partially grouted masonry, allowable stresses are determined on the basis of linear interpolation between fully grouted hollow units and ungrouted hollow units based on amount (percentage) of grouting.

The theoretical distribution of shear stress, f_v, along the length of the shear wall (Figure 3) for in-plane loads, or perpendicular to any wall for out-of-plane loads, is parabolic in shape for a rectangular cross-section. The calculated shear stress due to applied loads, f_v, as given by Equation 7 cannot exceed any of the code-prescribed allowable shear stresses, F_v, as follows:

a) $1.5\sqrt{f'_m}$ psi $\left(0.125\sqrt{f'_m} \text{ MPa} \right)$

b) 120 psi (827 kPa)

c) For running bond masonry not fully grouted:

    37 psi + $0.45 N_v/A_n$ $(255 + 0.45 N_v/A_n \text{ kPa})$

d) For masonry not laid in running bond, constructed of open-end units and fully grouted:

    37 psi + $0.45 N_v/A_n$ $(255 + 0.45 N_v/A_n \text{ kPa})$

e) For running bond masonry fully grouted:

    60 psi + $0.45 N_v/A_n$ $(414 + 0.45 N_v/A_n \text{ kPa})$

f) For masonry not laid in running bond, constructed of other than open-end units and fully grouted:
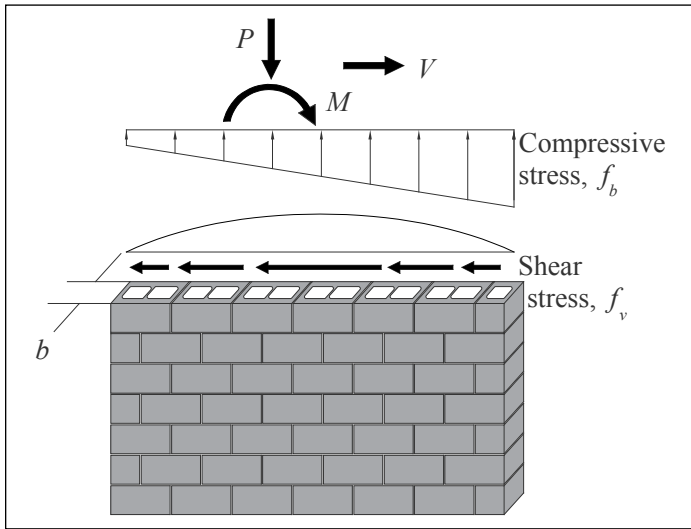
    15 psi (103 kPa)

Figure 3—Unreinforced Masonry Shear Walls

The MSJC Code defines the above allowable shear stresses as being applicable to in-plane shear stresses only: allowable shear stresses for out-of-plane loads are not provided. In light of this absence, *Commentary on Building Code Requirements for Masonry Structures* suggests using these same values for out-of-plane shear design.

REINFORCED MASONRY

Reinforced masonry design in accordance with the MSJC Code neglects the tensile resistance provided by the masonry units, mortar and grout in determining the strength of the masonry assemblage. Thus, for design purposes, the portion of masonry subjected to net tensile stresses is assumed to have cracked, transferring all tensile forces to the reinforcement. (While the determination of the reinforced masonry element strength conservatively assumes the portion of the masonry subjected to net tensile stresses has cracked, this should be verified when calculating the stiffness and deflection of a reinforced masonry element.)

Reinforcement

The tensile stress in the reinforcement due to applied load, f_s, is calculated as the product of the strain in the steel (which increases linearly in proportion to the distance from the neutral axis) and its modulus of elasticity, E_s. The modulus of elasticity, E_s, of mild steel reinforcement is assumed to be 29,000,000 psi (200 GPa). The code-prescribed allowable steel stresses are as follows (ref. 1a):

For Grade 60 bar reinforcement in tension:
F_s = 32,000 psi (220.7 MPa)

For Grade 40 and 50 bar reinforcement in tension:

F_s = 20,000 psi (137.9 MPa)

For wire joint reinforcement in tension:

F_s = 30,000 psi (206.9 MPa)

Unless ties or stirrups laterally confine bar reinforcement as required by the MSJC Code, the reinforcement is assumed not to contribute compressive resistance to axially loaded elements. When reinforcement is confined as prescribed, stresses are limited to the values listed above. Additional information on mild reinforcing steel can be found in TEK 12-04D, *Steel Reinforcement for Concrete Masonry* (ref. 7).

Reinforced Out-of-Plane Flexure

The allowable compressive stress in masonry, F_b, due to flexure or due to a combination of flexure and axial load is limited by Equation 8. When axial loads are not present, or are conservatively neglected as may be appropriate in some cases, there are several circumstances to consider in determining the flexural capacity of reinforced masonry assemblies.

$$f_b \leq F_b = 0.45\ f'_m \qquad \text{Eqn. 8}$$

For a fully grouted element, a cracked transformed section approach is used, wherein the reinforcement area is transformed to an equivalent area of concrete masonry using the modular ratio.

Partially grouted assemblies are analyzed in the same way, but with the additional consideration of the ungrouted cores. For partially grouted masonry there are two types of behavior to consider.

1. The first case applies when the neutral axis (the location of zero stress) lies within the compression face shell, as shown in Figure 2A. In this case, the masonry is analyzed and designed using the procedures for a fully grouted assembly.

2. The second type of analysis occurs when the neutral axis lies within the core area rather than the compression face shell, as shown in Figure 2B. For this case, the portion of the ungrouted cells must be deducted from the area of masonry capable of carrying compression stresses.

The neutral axis location depends on the relative moduli of elasticity of the masonry and steel, n, as well as the reinforcement ratio, ρ, and the distance between the reinforcement and extreme compression fiber, d.

When analyzing partially grouted assemblies, it is typically assumed that the neutral axis lies within the compression face shell, as the analysis is more straightforward. Based on this assumption, the resulting value of k and the location of the neutral axis (kd) is calculated. If it is determined that the neutral axis lies outside the compression face shell, the more rigorous tee beam analysis is performed. Otherwise, the rectangular beam analysis is carried out.

For design purposes, the effective width of the compression zone per bar is limited to the smallest of:

• six times the wall thickness,

• the center-to-center spacing of the reinforcement, or

• 72 in. (1,829 mm).

This requirement applies to masonry laid in running bond and to masonry not laid in running bond and containing bond beams

spaced no farther than 48 in. (1,219 mm) on center. Where the center-to-center spacing of the reinforcement does not control the effective width of the compression zone, the resulting resisting moment or resisting shear is proportioned over the width corresponding to the effective width of the compression zone as determined above.

Rectangular Beam Analysis

For fully grouted masonry elements and for partially grouted masonry elements with the neutral axis in the compression face shell, the resisting flexural capacity, M_r, is calculated as follows:

$$n = E_s/E_m \qquad \text{Eqn. 9}$$

$$\rho = \frac{A_s}{bd} \qquad \text{Eqn. 10}$$

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n \qquad \text{Eqn. 11}$$

$$j = 1 - k/3 \qquad \text{Eqn. 12}$$

$$M_m = \tfrac{1}{2} F_b k j b d^2 \qquad \text{Eqn. 13}$$

$$M_s = A_s F_s j d \qquad \text{Eqn. 14}$$

Where the resisting flexural capacity, M_r, is taken as the lesser of M_m and M_s.

Tee Beam Analysis

For partially grouted masonry assemblies where the neutral axis is located within the cores (i.e., when $kd > t_{fs}$), the resisting flexural capacity, M_r, is calculated using the neutral axis coefficient k given by Equation 15 and either Case A or Case B as follows:

$$k = \frac{-t_{fs}(b - b_w) - A_s n}{db_w} +$$
$$\frac{\sqrt{(t_{fs}(b - b_w) + A_s n)^2 + b_w(t_{fs}^2(b - b_w) + 2A_s n d)}}{db_w} \qquad \text{Eqn. 15}$$

(A) For cases where the masonry strength controls the design capacity:

$$f_s = nF_b\left(\frac{1-k}{k}\right) \qquad \text{Eqn. 16}$$

If f_s as determined using Equation 16 is greater than the allowable steel stress, F_s, then the steel controls the strength and the design is carried out using procedure (B) below. Otherwise, the internal compression force, C, and moment capacity are computed as follows:

$$C = \tfrac{1}{2} F_b b k d \qquad \text{Eqn. 17}$$

$$M_r = Cjd \qquad \text{Eqn. 18}$$

(B) For cases where the steel strength controls:

$$T = A_s F_s \qquad \text{Eqn. 19}$$

$$M_r = Tjd \qquad \text{Eqn. 20}$$

Reinforced Axial Compression

Axial loads acting through the axis of a member are distributed over the net cross-sectional area of the effective compression zone, or, for concentrated loads, $4t$ plus the bearing width. The allowable axial compressive force is based on the compressive strength of masonry and the slenderness ratio of the element in accordance with the following:

For elements having $h/r \le 99$, the allowable compressive force, P_a, is determined as follows:

$$P_a = (0.25 f'_m A_n + 0.65 A_s F_s)\left[1 - \left(\frac{h}{140r}\right)^2\right] \qquad \text{Eqn. 21}$$

For elements having $h/r > 99$, the allowable compressive force, P_a, is determined as follows:

$$P_a = (0.25 f'_m A_n + 0.65 A_s F_s)\left[\left(\frac{70r}{h}\right)^2\right] \qquad \text{Eqn. 22}$$

Note that compression reinforcement requires ties or stirrups to laterally confine the reinforcement.

Reinforced Axial Compression and Flexure

Often, loading conditions result in both axial load and flexure on a masonry element. Superimposing the stresses resulting from axial compression and flexural compression produces the combined stress. Members are proportioned so that this maximum combined stress does not exceed the allowable stress limitation imposed by Equations 5, 6, 8 and either Equation 21 or 22, as appropriate. In cases where the combined compressive stresses are relatively large, design economy may be realized by increasing the specified masonry compressive strength, f'_m, which in turn can result in thinner wall cross-sections, reduced material usage, and increased construction productivity. Several design approaches are available for combined axial compression and flexure, most commonly using either computer programs to perform the necessary iterative calculations or using interaction diagrams to graphically determine the required reinforcement for a given condition. One such software program is *EleMasonry Design Software* (ref. 9).

Reinforced Shear

Under the 2011 MSJC Code, the shear resistance provided by the masonry is added to the shear resistance provided by the shear reinforcement. This is a change from previous versions of the Code, and provides a better prediction of shear strength. Note that additional requirements apply to special reinforced masonry shear walls.

There are two checks to be made for reinforced shear. First, as for all ASD design, the calculated shear stress must be less than or equal to the allowable shear stress ($f_v \le F_v$). Secondly, when the calculated shear stress is greater than the allowable shear stress resisted by the masonry ($f_v > F_{vm}$), shear reinforcement must be provided. These calculations are presented below.

The applied shear stress on the masonry member is calculated as follows:

$$f_v = \frac{V}{A_{nv}} \qquad \text{Eqn. 23}$$

The allowable shear stress, F_v, is determined using Equation 24 and Equation 25 or 26, as appropriate.

$$F_v = F_{vm} + F_{vs} \qquad \text{Eqn. 24}$$

Where $M/Vd \leq 0.25$:

$$F_v \leq 3\sqrt{f'_m} \qquad \text{Eqn. 25}$$

Where M/Vd is ≥ 1.0:

$$F_v \leq 2\sqrt{f'_m} \qquad \text{Eqn. 26}$$

When the ratio M/Vd falls between 0.25 and 1.0, the maximum value of F_v may be linearly interpolated using Equations 25 and 26. The values of F_{vm} and F_{vs} are determined using Equations 27 and 28. When calculating F_{vm}, M/Vd must be taken as a positive number and need not exceed 1.

$$F_{vm} = \frac{1}{2}\left[\left(4.0 - 1.75\left(\frac{M}{Vd}\right)\right)\sqrt{f'_m}\right] + 0.25\frac{P}{A_n} \qquad \text{Eqn. 27}$$

$$F_{vs} = 0.5\left(\frac{A_v F_s d}{A_n s}\right) \qquad \text{Eqn. 28}$$

In addition, when $f_v > F_{vm}$, shear reinforcement must be provided in accordance with the following requirements:

• the shear reinforcement must be oriented parallel to the direction of the shear force,

• the shear reinforcement spacing must not exceed the lesser of $d/2$ or 48 in. (1,219 mm), and

• reinforcement must also be provided perpendicular to the shear reinforcement. This prescriptive reinforcement must have an area of at least one-third A_v, must be uniformly distributed, and may not be spaced farther apart than 8 ft (2,438 mm).

NOTATION

A_n = net cross-sectional area of a member, in.^{2} (mm^2)

A_{nv} = net shear area, in.^{2} (mm^2)

A_s = area of nonprestressed longitudinal reinforcement, in.^{2} (mm^2)

A_v = cross-sectional area of shear reinforcement, in.^{2} (mm^2)

b = width of section, in. (mm)

b_w = for partially grouted walls, width of grouted cell plus each web thickness within the compression zone, in. (mm)

C = resultant compressive force, lb (N)

d = distance from extreme compression fiber to centroid of tension reinforcement, in. (mm)

E_m = modulus of elasticity of masonry in compression, psi (MPa)

E_s = modulus of elasticity of steel, psi (MPa)

e = eccentricity of axial load, lb (N)

F_a = allowable compressive stress available to resist axial load only, psi (MPa)

F_b = allowable compressive stress available to resist flexure only, psi (MPa)

F_s = allowable tensile or compressive stress in reinforcement, psi (MPa)

F_v = allowable shear stress, psi (MPa)

F_{vm} = allowable shear stress resisted by the masonry, psi (MPa)

F_{vs} = allowable shear stress resisted by the shear reinforcement, psi (MPa)

f_a = calculated compressive stress in masonry due to axial load only, psi (MPa)

f_b = calculated flexural bending stress in masonry, psi (MPa)

f'_m = specified compressive strength of masonry, psi (MPa)

f_s = calculated tensile or compressive stress in reinforcement, psi (MPa)

f_v = calculated shear stress in masonry, psi (MPa)

h = effective height of masonry element, in. (mm)

I_n = moment of inertia of net cross-sectional area of a member, in.^{4} (mm^4)

j = ratio of distance between centroid of flexural compressive forces and centroid of tensile forces to depth, d

k = ratio of distance between compression face of element and neutral axis to the effective depth d

M = maximum calculated bending moment at section under consideration, in.-lb, (N-mm)

M_m = flexural strength (resisting moment) when masonry controls, in.-lb (N-mm)

M_r = flexural strength (resisting moment), in.-lb (N-mm)

M_s = flexural strength (resisting moment) when reinforcement controls, in.-lb (N-mm)

N_v = compressive force acting normal to shear surface, lb (N)

n = modular ratio, E_s/E_m

P = axial compression load, lb (N)

P_a = allowable axial compressive force in a reinforced member, lb (N)

P_e = Euler buckling load, lb (N)

Q = first moment of inertia about the neutral axis of an area between the extreme fiber and the plane at which the shear stress is being calculated, in.^{3} (mm^3)

r = radius of gyration, in. (mm)

s = spacing of shear reinforcement, in. (mm)